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Estimation of battery available capacity under variable discharge currents

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Abstract

In this paper, a new mathematical model in semi-empirical form for lead-acid batteries is presented, which describes the relationship between the battery terminal voltage and the variable discharge current. Based on the proposed model, a new estimation method of the battery available capacity (BAC) in the presence of variable discharge currents is developed. The method involves the real-time identification of the model parameters which are then used to estimate the BAC according to the predefined cutoff voltage and the trend of battery terminal voltage during discharging. Thus, both temperature and aging influences on the BAC are considered inherently. Comparisons between the calculated results and the measured data confirm that the proposed method can provide an accurate real-time estimation of the BAC under variable discharge currents. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Battery capacity estimation; Battery available capacity; Lead-acid battery; Simulated discharge pattern; Variable discharge current

1. Introduction

With the growing concerns over the environmental protection and energy conservation in recent years, research and development on the improvement of various electric vehicle (EV) technologies are being actively conducted [1,2]. Nevertheless, the development of EV batteries cannot keep pace with that of other EV technologies [3]. The key shortcomings of the available EV batteries are very limited energy storage and highly nonlinear characteristics. Thus, the battery available capacity (BAC) indicator becomes a crucial device for EVs.

Traditionally, the estimation of the BAC under variable discharge currents has been represented by the estimation of the battery state of charge (SOC). There are countless papers describing various attempts to estimate the SOC using various computational approaches, which are summarized in [4]. However, it is worth emphasizing that the BAC, in most cases, is different from the SOC. Although the higher the SOC, the greater the BAC can be exhibited at the same discharge current regime, there is no quantitative relationship between them. Some direct estimation methods of the BAC have ever been explored. For the constant discharge current, the BAC of lead-acid batteries can be estimated empirically based on the Peukert equation. Recently, the estimation accuracy of the BAC has been significantly improved using the artificial neural network model [5]. For the variable discharge current, the published methods for the calculation of the BAC can be categorized into two groups — either based on the average discharge current [6,7] or based on the reference discharge current [8,9]. For the former, unless the discharge currents do not vary significantly, the discharge current regimes cannot be simply represented by the average discharge current. Table 1 shows a comparison of the BACs under different discharge current regimes, where the controlled temperature is 25°C, the specified minimum voltage is 10.8 V and the battery is fully charged before discharge. It can be found that although the average discharge currents corresponding to the US federal urban driving schedule (FUDS), the US federal highway driving schedule (FHDS) and the European reference driving cycle (ECE) for EVs are all approximately equal to 13 A (about the 3 h discharge rate for the nominal capacity of $C_N = 40$ Ah), their BACs are very different. The reason is due to the fact that the BAC is strongly influenced by the discharge current regime. For the latter, the constant capacity at the reference discharge current is taken as the BAC. If the discharge current is higher or lower than the reference discharge current, the corrective coefficients derived from the constant current discharge will be used to estimate the equivalent BAC. Under this condition, the effect of the discharge current on the BAC at different SOCs will be ignored, leading to create significant errors as shown in Table 2, where the experimental conditions are the same as those above-mentioned for EV driving cycles. Furthermore,

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Table 1
BAC under different discharge current regimes

Discharge current regimes	Average discharge current (A)	BAC (Ah)
FUDS	13.08	15.96
FHDS	13.11	25.05
ECE	13.21	13.05

Table 2
BAC under different discharge current profiles

Discharge current profiles	BAC (Ah)
8 A for first 3 h and 20 A for other 0.38 h	31.66
20 A for first 0.38 h and 8 A for other 3.17 h	33.00
8 A for first 3.17 h and 20 A for other 0.1 h	27.33

these corrective coefficients cannot be real-time updated in the presence of the battery aging.

The purpose of this paper is to propose a new estimation method for determining the BAC for EVs, in which the variable discharge current regimes as well as the temperature and aging influences are taken into account. Also, the proposed method will be real-time implemented and experimented. Comparisons of the calculated results and the measured data will be conducted to verify the accuracy and practicability of the proposed method.

2. Battery available capacity

2.1. Definition

Generally, the BAC refers to the quantity of electricity that can be delivered at a certain discharge current regime until the predefined criterion for the end of discharge. Three possible criteria for EVs were mentioned in [10]. The first is a specified minimum terminal voltage. The second is a power specified by the EV standard driving cycles at which the battery can no longer deliver. The third is a specified reference capacity. The first criterion is adopted in this study because it is most operable and controllable from the experimental and practical point of view. Also, the second and third criteria are essentially the same as the first criterion. For instance, the high acceleration rate in the early stage of the FUDS constitutes the maximum power demand and, thus, the maximum discharge current. When this peak power cannot be met, according to the second criterion, all its BAC has already been delivered. At this moment, the battery terminal voltage also drops rapidly. In order to protect the battery from dropping to an unreasonable voltage level, the minimum terminal voltage needs to be specified (hence the first criterion). A similar deduction is also valid for the third criterion.

In this paper, the BAC value C_a is defined by the specified minimum terminal voltage V_{min} criterion, and governed by the discharge current regime $i(t)$, the battery surface temperature $T_s(t)$ and aging $K_{age}(t)$. Therefore, the BAC can be estimated when the battery is discharged at a certain discharge current regime until the specified minimum terminal voltage V_{min} is reached, namely,

$$C_a = f(i(t), V(t), T_s(t), K_{age}(t))|_{V(t)=V_{min}} \quad (1)$$

where $V(t)$ is the instantaneous battery terminal voltage. From this definition, the BAC can be estimated by various methods under variable discharge currents.

2.2. Possible estimation methods

The first method is to find a certain constant value K which has a relationship with the BAC regardless of the variation of discharge currents and is similar to the K of Peukert equation for the constant discharge current. This relationship can be expressed as

$$K = g(i(t), C_a) \quad (2)$$

However, this constant value K may not exist for variable discharge currents. Table 2 shows a comparison of BAC under different discharge current profiles. It can be found that the BAC is altered by the change of discharge current profiles. Thus, a constant value of K is hardly possible to be identified. Even if a constant value is approximately found for variable discharge currents, the battery aging effect on the K is still a drawback.

The second method is to find the battery voltage model that can describe accurately various phenomena inside the battery including the relationship between the battery terminal voltage and the BAC. Then, the predefined discharge current regimes are used to discharge this model until the predefined minimum terminal voltage is reached. As a result, the corresponding BAC can be obtained. A lot of efforts to describe the battery terminal voltage, the BAC and the discharge current were made. Among them, the Shepherd model [11] is widely accepted. The parameters of the Shepherd model are calculated from the experimental curves for the constant current discharge. However, there are three shortcomings of the Shepherd equation. First, the battery model parameters can only be calculated from the experimental discharge curves with the constant current. Second, the applicability of this model appears to be limited to temperature range from 10 to 30°C. Third, the aging effect on the battery model parameters has not been considered. For the first two problems, it is almost impossible for the battery to satisfy these requirements under EV operations. For the last one, it is the toughest problem for almost all mathematical expressions or equivalent electric circuits based on the theory of electrochemistry.

The third method is to do a lot of experiments under those different discharge current regimes that the battery may encounter under EV operation. Then the BAC for these discharge current regimes can be obtained. Table 3 shows the resulting BACs and their percentages with respect to the nominal capacity C_N under the normal distribution discharge current (NDDC) with the average current of 14 A and the

Table 3
BAC and percentage of nominal capacity under different discharge current regimes

Discharge current regimes	BAC (Ah)	C_a/C_N (%)
NDDC	25.75	64.38
UDDC	20.13	50.33
FUDS	15.96	39.90
FHDS	25.05	62.63
ECE	13.05	32.63

standard deviation of 5 A and the uniform distribution discharge current with the mean current of 20 A between 12 and 28 A (UDDC) as well as the discharge current profiles corresponding to the FUDS, FHDS and ECE. This method is valid provided that the following two assumptions are correct. First, the same type of battery has the same BAC at the same discharge current regime. Second, the condition of the battery under test (mainly referring to the temperature) is the same as that of the battery used for EV operation. Obviously, these two requirements are not easy to be satisfied in EVs. The major disadvantage is that this method is ill-suited for any other discharge current regimes different from those included in Table 3. Also, the effect of aging on the BAC has not been taken into account.

3. Proposed method

3.1. Battery voltage model

The proposed mathematical model of the battery voltage $V(t)$ under the variable discharge current $i(t)$ is expressed as

$$V(t) = V_0(t) + V_1(t)\log\left(1 - \beta(t)\frac{q(t)}{C_N}\right) - R(t)i(t) \quad (3)$$

where the first two terms can be derived from the Nernst equation [12] for the lead-acid batteries as given by

$$E = E_0 + E_1 \log\left(\frac{a_{H_2SO_4}}{a_{H_2O}}\right) \quad (4)$$

which describes the gradual decrease of the battery voltage with the decrease of the electrolyte concentration during discharging. Both the $V_0(t)$ and $V_1(t)$ denote the voltage coefficients. The $R(t)$ is the battery internal resistance including the electrode, electrolyte and wire resistance as well as the average equivalent resistance which leads to the activation and concentration overpotential. The $\beta(t)$ is the decaying factor on the utilization of battery active materials due to the high discharge rate, as illustrated by Table 3. The $q(t)$ is the discharged capacity with respect to a certain discharge current regime, which can be expressed as

$$q(t) = \int_{t_0}^{t} i(t)\,dt \quad (5)$$

where t_0 is the starting time of the discharge.

3.2. Experimental results and parameter identification

Battery testing plays an important role in evaluating the performance of batteries, especially for those batteries used in EVs. Fig. 1 shows the battery evaluation and testing system. This system consists of four main parts as follows.

1. A programmable charger in which almost any charging algorithms can be performed.
2. A programmable electronic load in which flexible and variable discharge current regimes can be designed, such as the constant current discharge, constant power discharge

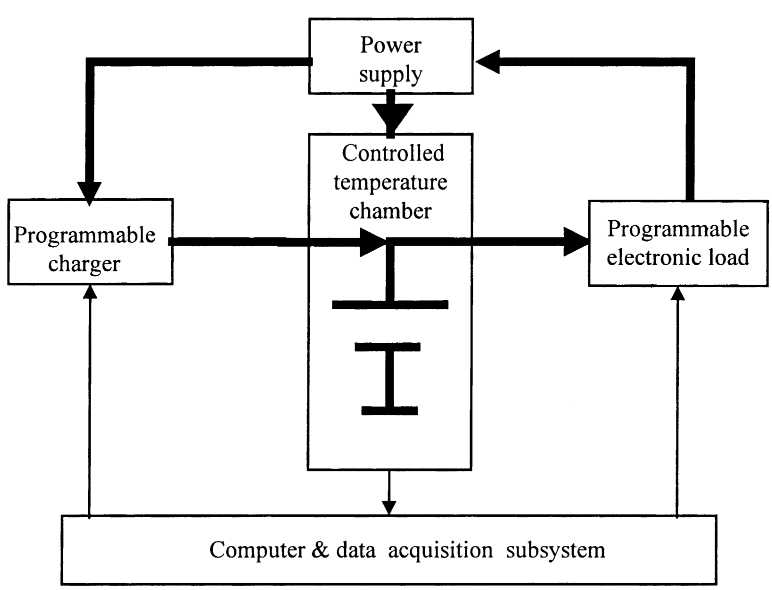

Fig. 1. Battery evaluation and testing system.

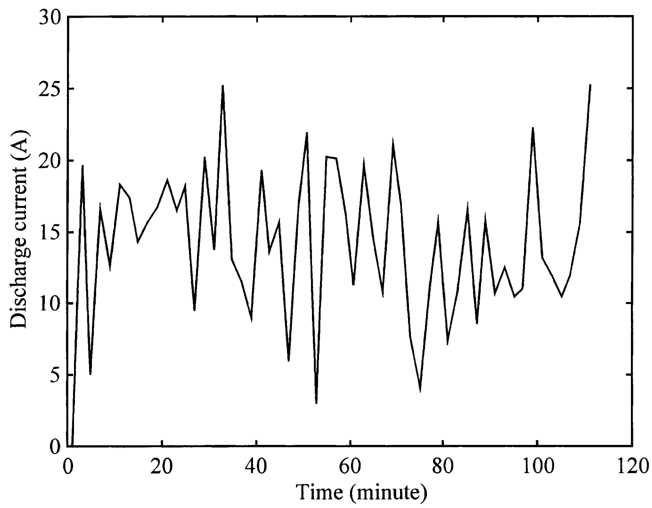

Fig. 2. Discharge current regime for NDDC operation.

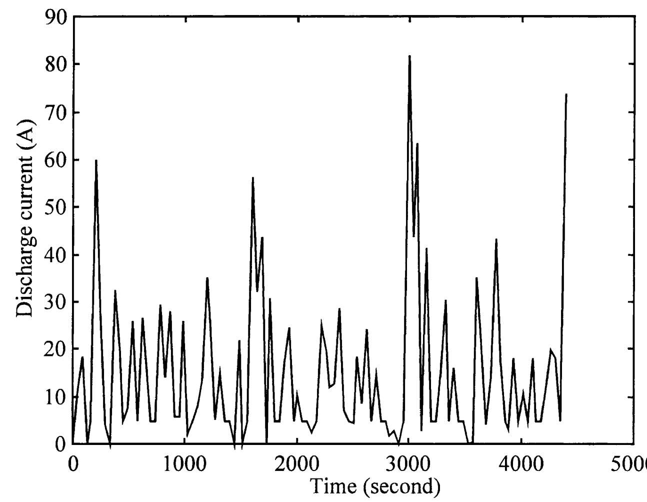

Fig. 4. Discharge current regime for FUDS operation.

and constant resistance discharge as well as the discharge currents corresponding to various driving cycles.

3. A temperature controlled chamber in which batteries can be tested under any predefined air temperature over the range from −20 to 50°C.
4. A data acquisition subsystem in which the sampling time can be preset as in the order of seconds, minutes or hours, depending on the user's requirements.

With this system, the batteries can be tested under different charge or discharge currents, and temperatures, hence simulating the battery operation in EVs.

Normally, from the experimental point of view, the estimation of the BAC can be carried out under two conditions. First, the theoretical discharge current regime is used to discharge the battery until the battery terminal voltage reaches the specified minimum voltage. For example, the discharge current regimes for the NDDC and UDDC operations are, respectively, shown in Figs. 2 and 3. Second, the EV discharge current regime is used to discharge the battery until the battery terminal voltage is decreased to the specified minimum voltage. For example, the discharge current regimes for the FUDS, FHDS and ECE operations are, respectively, shown in Figs. 4–6. For these two types of discharge current regimes, all the experiments for lead-acid batteries are carried out with the specified minimum voltage of 10.8 V for the nominal voltage of 12 V. During testing, both the battery terminal voltage and the discharge current are measured, and the discharged capacity is calculated at each sampling period. Then Eq. (3) is used to describe the battery terminal voltage under variable discharge currents. The parameters in Eq. (3) are identified by using the nonlinear least square method.

Considering that $V(t)$, $q(t)$ and $i(t)$ are sampled at each minute or second for a certain test, the time T_0 is the calculation period and the whole discharge period is t_{end}. According to Eq. (3), the nonlinear least square method can

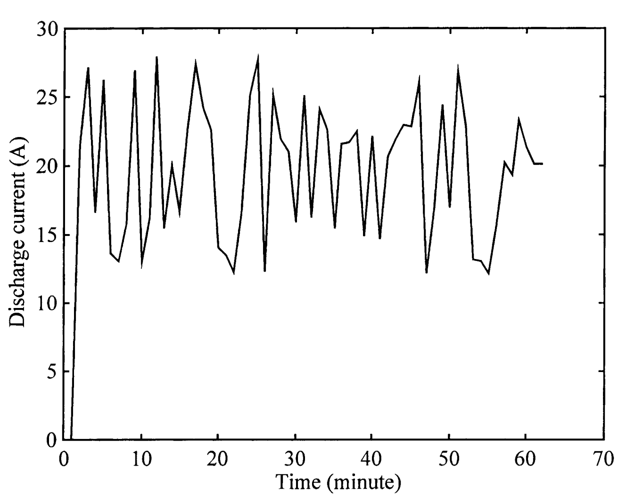

Fig. 3. Discharge current regime for UDDC operation.

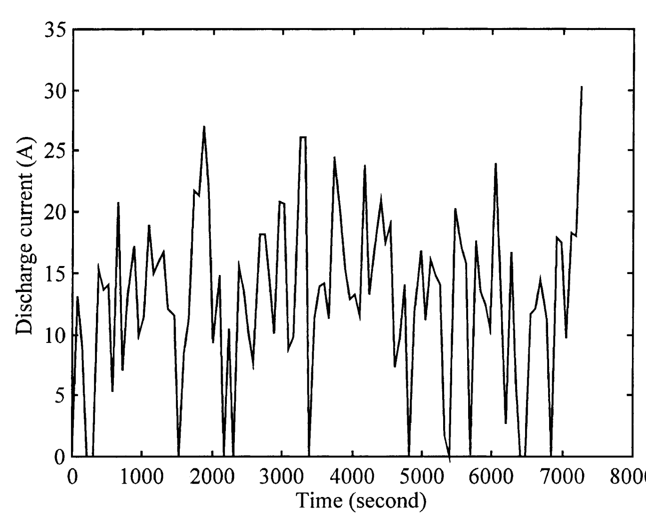

Fig. 5. Discharge current regime for FHDS operation.

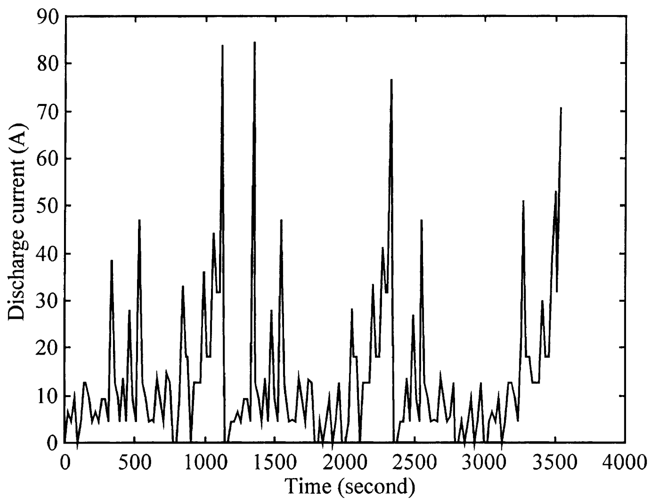

Fig. 6. Discharge current regime for ECE operation.

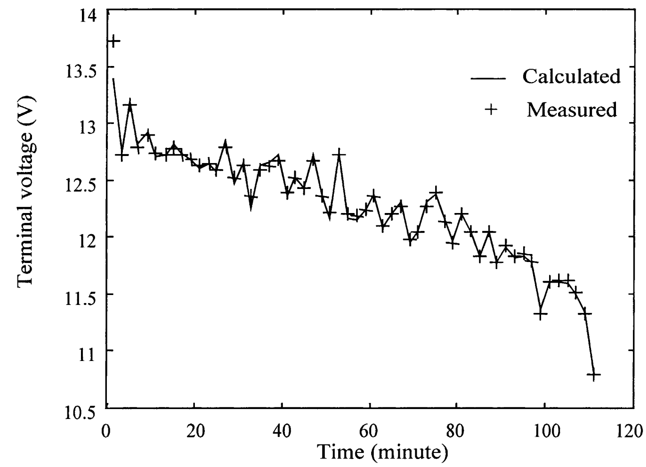

Fig. 7. Terminal voltage comparison for NDDC operation.

be realized by

$$\min F(X) = \min\sum_{j=1}^{n} \times \left[V(jT_0) - \left(V_0(T) + V_1(T)\log \times \left(1 - \beta(T)\frac{q(jT_0)}{C_N}\right) - R(T)i(jT_0)\right)\right]^2 \qquad (6)$$

where $T = nT_0$ $(n = 1, 2, 3, \ldots, n_{\max})$, $n_{\max}T_0 = t_{\text{end}}$, and $X = [V_0(T),\ V_1(T),\ \beta(T),\ R(T)]$. The initial values of $X = [13, 1, 1, 0.5]$ are taken between the upper limit $[15, \infty, 3, \infty]$ and the lower limit $[10, 0, 1, 0.0001]$. Using the optimization toolbox of MATLAB, Eq. (6) can be solved and the parameters of Eq. (3) at the end of discharge are listed in Table 4. Then, these parameters are used to calculate the battery terminal voltage. As shown in the Figs. 7–11, the calculated terminal voltage closely agrees with the measured terminal voltage for different operating conditions, including the NDDC, UDDC, FUDS, FHDS and ECE. This result shows that Eq. (3) can accurately describe the relationship between the battery terminal voltage and the variable discharge current. Inspired by this result, the new real-time estimation method based on Eq. (3) under variable discharge currents is proposed below.

Table 4
Parameters at end of discharge under different discharge current regimes

Discharge current regimes	$V_0(t_{\text{end}})$ (V)	$V_1(t_{\text{end}})$ (V)	$\beta(t_{\text{end}})$	$R(t_{\text{end}})$ (Ω)
NDDC	13.38723	1.200556	1.1368	0.031413
UDDC	13.38359	0.600739	1.8947	0.0302
FUDS	13.28668	0.805015	1.5212	0.022307
FHDS	13.29248	1.088844	1.1346	0.026696
ECE	13.28031	0.449033	2.3859	0.021854

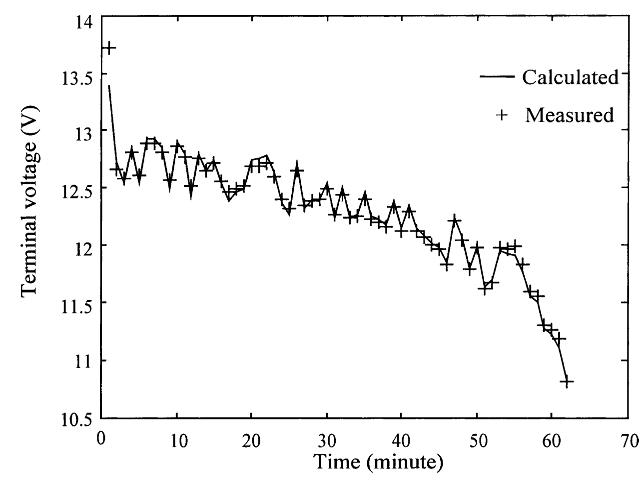

Fig. 8. Terminal voltage comparison for UDDC operation.

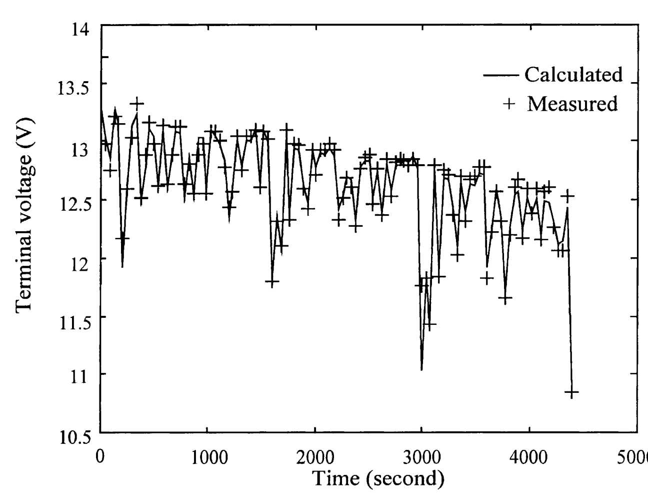

Fig. 9. Terminal voltage comparison for FUDS operation.

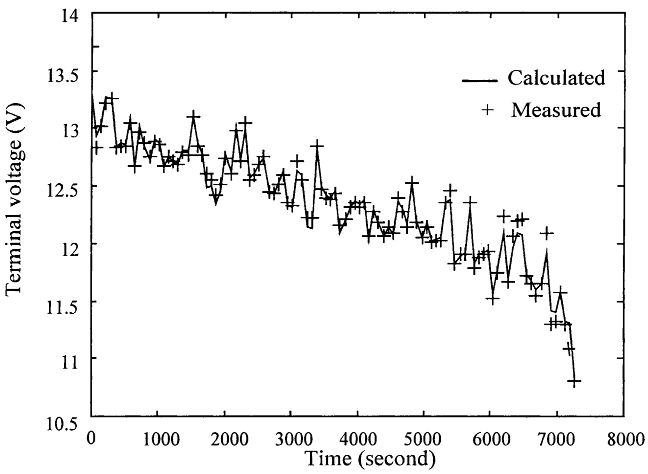

Fig. 10. Terminal voltage comparison for FHDS operation.

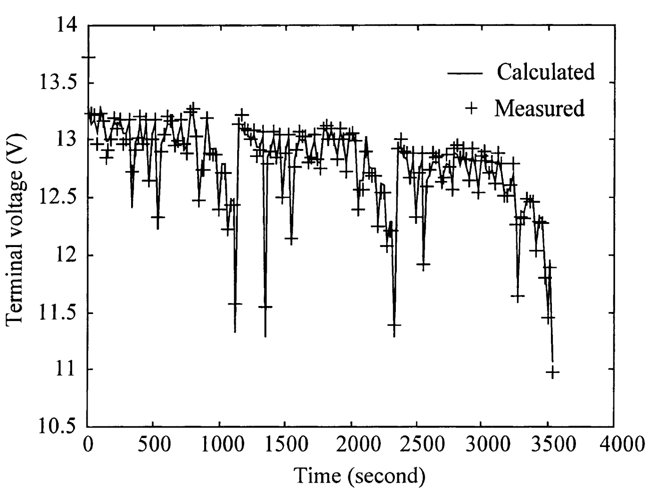

Fig. 11. Terminal voltage comparison for ECE operation.

3.3. Real-time estimation of BAC

From (3), the battery terminal voltage changes frequently with the variation of discharge currents. If this battery voltage model is directly used to estimate the BAC, obviously, it cannot work. However, the variation of the battery terminal voltage, indeed, has a remarkable feature that it decreases gradually throughout the discharge process and can be regarded as the superposition of two components. One component is the alternating part, where the terminal voltage varies with the change of the discharge current. The other component is similar to the situation under the constant current discharge, where the terminal voltage decreases gradually with the increase of the discharged capacity. So, the upper and lower boundaries, namely, the $V_u(t)$ and $V_l(t)$, of the terminal voltage are expressed as

$$V_u(t) = V_0(t) + V_1(t)\log\left(1 - \beta(t)\frac{q(t)}{C_N}\right) \quad (7)$$

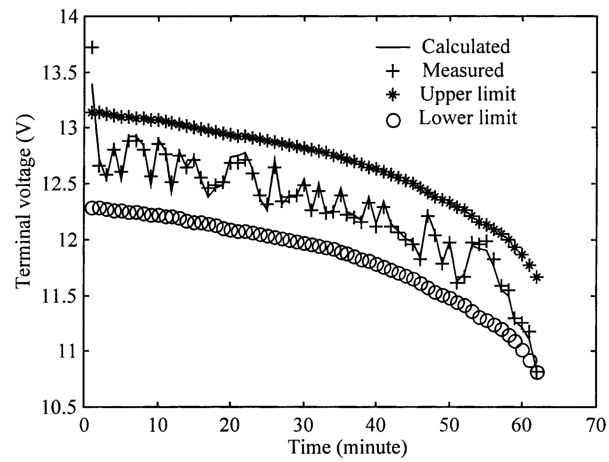

Fig. 12. Trend of terminal voltage variation for UDDC operation.

$$V_l(t) = V_0(t) + V_1(t)\log\left(1 - \beta(t)\frac{q(t)}{C_N}\right) - R(t)i_{max}(t) \quad (8)$$

where $i_{max}(t)$ denotes the instantaneous maximum value of the discharge current starting from the beginning of the discharge. Figs. 12 and 13 show the upper and lower boundaries of the terminal voltage under UDDC and ECE operations, respectively. It can be found that the $V_l(t)$ has the same trend as the $V(t)$ to go to the specified minimum voltage. Moreover, they both reach the specified minimum voltage almost simultaneously for the complete discharge process. Therefore, Eq. (8) can be used to estimate the BAC when the voltage $V_l(t)$ decreases to the specified minimum voltage, namely $V_{min} = 10.8$ V, and can be rearranged as

$$C_a(T) = q(t)|_{V_l(t)=V_{min}} = \frac{C_N(1 - \exp((V_{min} + R(T)i_{max}(t) - V_0(T))/V_1(T)))}{\beta(T)} \quad (9)$$

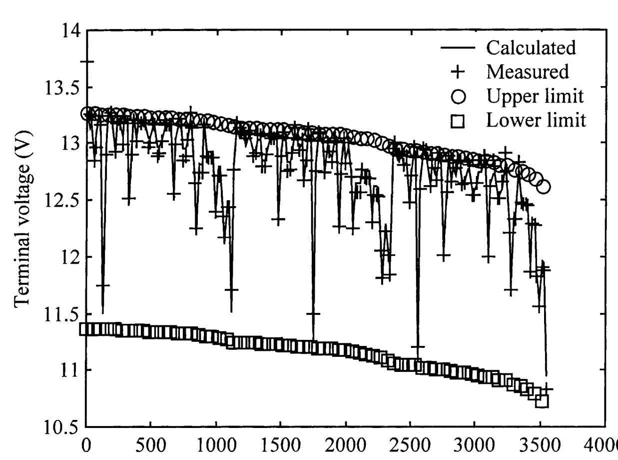

Fig. 13. Trend of terminal voltage variation for ECE operation.

Using Eq. (9) and the parameters identified from Eq. (6), the BAC can be estimated in the real-time fashion. The realization of the proposed method is summarized as follows.

- Define the discharge current regime whose average discharge current is about the 3 h discharge rate for the given battery.
- Discharge the battery at the fully charged state using the above defined discharge current regime.
- Measure the battery terminal voltage and discharge current, and calculate the discharged capacity at each sampling period.
- Identify the parameters in Eq. (3) that is given by Eq. (6) at each calculation period.
- Estimate the BAC using Eq. (9) with the parameters obtained from Eq. (6) at each calculation period.
- Compare the available capacities at each calculation period until it satisfies

$$|C_a((k+1)T_0) - C_a((k+2)T_0)| \leq |C_a(kT_0) - C_a((k+1)T_0)| \leq \delta \qquad (10)$$

where $\delta = 0.5$ Ah. It indicates that the trend of the battery terminal voltage for the given discharge current regime is formed. Thus, the $C_a((k+2)T_0)$ is regarded as the BAC for this discharge current regime.

According to the procedure of realization mentioned above, the BACs for the NDDC, UDDC, FUDS, FHDS and ECE operations at each 5 min interval are calculated and plotted in Figs. 14 and 15. It can be observed that the corresponding BACs are approaching 27.36, 19.94, 16.02, 25.91 and 13.55 Ah, respectively. Compared with the experimental data (25.75, 20.13, 15.96, 25.05, and 13.05 Ah) listed in Table 3, the agreement is very acceptable, especially under variable discharge current regimes.

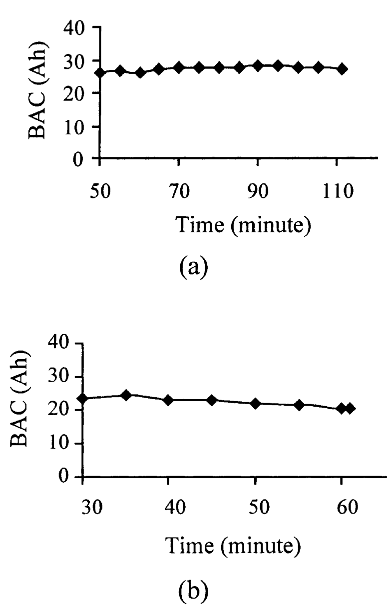

Fig. 14. Calculated BAC under theoretical discharge current regimes: (a) NDDC; (b) UDDC.

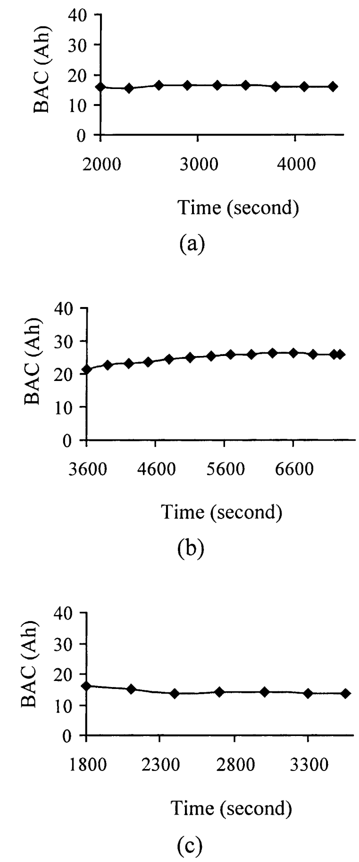

Fig. 15. Calculated BAC under EV discharge current regimes: (a) FUDS; (b) FHDS; (c) ECE.

It has been shown that the BAC under the variable discharge currents can be calculated by the real-time identification of the parameters in the battery voltage model. So it overcomes the aging and temperature influences on the BAC automatically. Furthermore, Eq. (3) can be considered as a general model for the secondary batteries. Thus, the proposed method can be easily extended to the estimation of the BAC of other types of EV batteries, such as the nickel cadmium and nickel metal hydride ones.

Since the BAC is estimated in real time by using the predefined cutoff voltage and the trend of battery terminal voltage, the proposed method in principle can also be used to estimate the BAC for EV batteries in the regenerative charge condition. Detailed investigation and experimentation of EV batteries under this condition will be discussed in the later paper.

4. Conclusions

In this paper, a new mathematical model for lead-acid batteries is proposed. Based on this model, a new estimation method of the BAC under variable discharge currents is developed and implemented by the real-time identification of the model parameters. Thus, both the temperature and aging influences on the BAC has been taken into account inherently. The comparison between the calculated results and the experimental data confirms that the proposed method can provide a good estimation of the BAC under variable discharge currents for EV operations. Furthermore, this method can readily be extended to estimate the BAC of other batteries.

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